# Mathematical Analysis OF Quick Draw Hold'em 

Prepared

For

John Feola
New Vision Gaming 5 Samuel Phelps Way North Reading, MA 01864

Office 978-664-1515
Fax 978-664-5117
www.newvisiongaming.com

## By

Stanley Ko
GAMBOLOGY
P. O. Box 82225

Las Vegas, NV 89180
702-258-9685

April 21, 2006

Quick Draw Hold'em is a new table game similar to Hold'em Poker. It is played with one standard deck of cards. The player does not compete against the dealer.

## Rules of Play

Quick Draw Hold'em offers a primary wager and two separate optional bonus side bets. The first bonus side bet is the 3 Card Flop Bonus; the second bonus side bet is based on the Best 4 out of 5 Community Cards. Players win according to pay tables for each of the Bonus side bets. Bonuses will be paid on the highest hand only.

To start the game, players must place a primary wager in the circle marked Flop. The optional Bonus bets must also be placed at the start of each game. The dealer beginning from their left will deal each player who has made the Flop bet two cards face down. The dealer will then place five community cards in the appropriate spots on the table.

The dealer will reveal the first three community cards (known as the flop). Nonwinning 3 Card Flop Bonus bets will be collected by the dealer. Winning 3 Card Flop Bonus Bets will be paid by the dealer according to the 3 Card Flop Bonus pay tables below:

| Hand Type | B1 | B2 | B3 |
| :--- | ---: | ---: | ---: |
| Straight Flush | 40 | 40 | 40 |
| 3 of a Kind | 30 | 25 | 30 |
| Straight | 6 | 6 | 6 |
| Flush | 4 | 4 | 3 |
| One Pair | 1 | 1 | 1 |

All payouts are "to 1."
After players view their two cards and the first three Community cards, they have the option to either fold or place an additional wager equal to the Flop wager in the circle marked Turn. The dealer will then reveal the $4^{\text {th }}$ community card (known as the Turn Card). Players who fold will lose their Flop bet and the dealer will collect all the Flop bets of all those players who have elected to fold.

After players view the Turn card, they have an option to either fold or place an additional wager equal to the Turn wager in the circle marked River Bonus. Players who fold will lose their wagers placed on both the Flop bet and Turn bet and the dealer will collect wagers from each player who has folded their hand. The dealer will then reveal the $5^{\text {th }}$ and final community card (known as the River card).

Players who have made the River Bonus Bet will have their 2 card hand combined with the 5 community cards. Players who achieve a winning hand using the best 5 out of 7 cards will be paid on all three of the following betting spots as follows:

- Flop Bet even money
- Turn Bet even money
- River Bonus Bet will be paid according to the River Bonus pay table. The pay tables are shown below:

| Hand | A1 | A2 | A3 |
| :--- | ---: | ---: | ---: |
| Royal Flush | 250 | 250 | 200 |
| Straight Flush | 50 | 50 | 50 |
| 4 of a Kind | 20 | 15 | 25 |
| Full House | 5 | 5 | 5 |
| Flush | 3 | 4 | 4 |
| Straight | 2 | 3 | 3 |
| 3 of a Kind | 1 | 1 | 2 |
| 7s-High 2 Pair (e.g., 7,7,4,4; <br> J, J,2,2 but not 6,6,3,3) |  | 1 |  |

## All payouts are "to 1."

Players who do not have a 7 card hand that is 7 s-high two pair or better will have all their 3 wagers collected by the dealer.

Players who have placed a bonus bet on the Best 4 out of 5 Community Cards are still eligible to receive their bonus even if they elect to fold during the core game. There are four-card straights, flushes and straight flushes but there will be no full houses. The pay tables are as follows:

| Hand Type | C1 | C2 | C3 | C4 | C5 | C6 |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| Royal Flush | 500 | 500 | 400 | 250 | 250 | 250 |
| 4 of a Kind | 100 | 100 | 80 | 100 | 100 | 100 |
| Straight Flush | 50 | 30 | 25 | 40 | 30 | 50 |
| 3 of a kind | 5 | 5 | 5 | 6 | 6 | 5 |
| Flush | 3 | 3 | 3 | 3 | 3 | 3 |
| Straight | 3 | 3 | 3 | 3 | 3 | 3 |
| Two Pair | 2 | 2 | 2 | 2 | 2 | 2 |
| A Pair of Jacks O/B | 1 | 1 | 1 | 1 | 1 | 1 |

All payouts are "to $1 . "$

## Mathematical Analysis

There are $C_{52,5} \times C_{47,1} \times C_{46,1}=2,598,960 \times 47 \times 46=56,189,515,200$ ways to deal five cards from a 52-card deck, one card from the remaining 47-card deck and then one card from the remaining 46-card deck. A computer program known as a combinatorial analyzer was developed to evaluate all of the hands in order to determine the optimal strategies and the player expectation. The algorithm for the analysis is described below.

First of all, an interim pay table is devised. Then the analyzer cycles through all 134,459 distinct 5 -card hands ${ }^{1}$. It calculates the return for each of the 5card hands by considering all possible 10812 -card combinations of the remaining 47 cards, and notes down the return for each of 476 -card hands. (Although each of the 10812 -card combinations from the 47-card deck can be dealt in two ways, the return for the entire 7 -card hand remains the same regardless of the order of the 2 cards. So, it's unnecessary to calculate the return twice. However, the order is only considered when determining the strategy and return for the 6 -card hands.) The return is calculated under the assumption that the player has not folded. If the return for a 6-card hand is less than -2 , the 6 -card hand should be folded. If the return for a 5 -card hand is less than -1, the 5 -card hand should be folded. The play (to press it up or fold) that results in the better return is selected for every hand being evaluated.

Once the strategy for the 5 -card hands has been determined, the player's overall expectation is derived. Depending on whether the expectation is acceptable, the interim pay table is tweaked and the same process is repeated until the player expectation derived is satisfactory.

The 5-card hand optimal strategy for pay table A3 is given in Appendix A.

[^0]
## Pay Table A1

Final Hand Probability Distribution and Expectations

| Hand | \% 2 Bets | $\%$ 3 Bets | Pays | \% Probability | \% Return |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: |
| Royal Flush | 0 | 0.002925 | 250 | 0.002925 | 0.7370 |
| Straight Flush | 0 | 0.025444 | 50 | 0.025444 | 1.3231 |
| 4 of a Kind | 0 | 0.166641 | 20 | 0.166641 | 3.6661 |
| Full House | 0 | 2.557596 | 5 | 2.557596 | 17.9032 |
| Flush | 0 | 2.450435 | 4 | 2.450435 | 14.7026 |
| Straight | 0.241582 | 3.204102 | 3 | 3.445684 | 15.5373 |
| 3 of a Kind | 0 | 4.199194 | 2 | 4.199194 | 16.7968 |
| 7s-High 2 Pair | 0 | 17.676716 | 1 | 17.676716 | 53.0301 |
| Nothing | 2.536616 | 27.346451 | -1 | 29.883067 | -87.1126 |
| Fold 1 ${ }^{\text {st }}$ Bet | 0 | 0 | -1 | 39.592298 | -39.5923 |
| Total | 2.778198 | 57.629504 |  | 100.000000 | -3.0086 |

The hit frequency is $30.2831 \%$. The player's average bet will be 2.1804 bets. The house advantage is $3.0086 \%$ of the ante or 3.0086\% / $2.1804=1.3799 \%$ per total wager.

## Pay Table A2

Final Hand Probability Distribution and Expectations

| Hand | \% 2 Bets | \% 3 Bets | Pays | \% Probability | \% Return |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: |
| Royal Flush | 0 | 0.002925 | 250 | 0.002925 | 0.7370 |
| Straight Flush | 0 | 0.025444 | 50 | 0.025444 | 1.3231 |
| 4 of a Kind | 0 | 0.166641 | 15 | 0.166641 | 2.8329 |
| Full House | 0 | 2.557596 | 5 | 2.557596 | 17.9032 |
| Flush | 0 | 2.450435 | 4 | 2.450435 | 14.7026 |
| Straight | 0.241582 | 3.204102 | 3 | 3.445684 | 15.5373 |
| 3 of a Kind | 0 | 4.199194 | 2 | 4.199194 | 16.7968 |
| 7s-High 2 Pair | 0 | 17.676716 | 1 | 17.676716 | 53.0301 |
| Nothing | 2.536616 | 27.346451 | -1 | 29.883067 | -87.1126 |
| Fold 1 ${ }^{\text {st }}$ Bet | 0 | 0 | -1 | 39.592298 | -39.5923 |
| Total | 2.778198 | 57.629504 |  | 100.000000 | -3.8419 |

The hit frequency is $30.2831 \%$. The player's average bet will be 2.1804 bets. The house advantage is $3.8419 \%$ of the ante or 3.8419\% / 2.1804 = 1.7620\% per total wager.

## Pay Table A3

Final Hand Probability Distribution and Expectations

| Hand | \% 2 Bets | $\%$ 3 Bets | Pays | \% Probability | \% Return |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: |
| Royal Flush | 0 | 0.002925 | 200 | 0.002925 | 0.5908 |
| Straight Flush | 0 | 0.025444 | 50 | 0.025444 | 1.3231 |
| 4 of a Kind | 0 | 0.166641 | 25 | 0.166641 | 4.4993 |
| Full House | 0 | 2.557596 | 5 | 2.557596 | 17.9032 |
| Flush | 0 | 2.450435 | 4 | 2.450435 | 14.7026 |
| Straight | 0.241582 | 3.204102 | 3 | 3.445684 | 15.5373 |
| 3 of a Kind | 0 | 4.199194 | 2 | 4.199194 | 16.7968 |
| 7s-High 2 Pair | 0 | 17.676716 | 1 | 17.676716 | 53.0301 |
| Nothing | 2.536616 | 27.346451 | -1 | 29.883067 | -87.1126 |
| Fold 1 ${ }^{\text {st }}$ Bet | 0 | 0 | -1 | 39.592298 | -39.5923 |
| Total | 2.778198 | 57.629504 |  | 100.000000 | -2.3217 |

The hit frequency is $30.2831 \%$. The player's average bet will be 2.1804 bets. The house advantage is $2.3217 \%$ of the ante or 2.3217\% / 2.1804 = 1.0648\% per total wager.

## 3 Card Flop Bonus Side Bet

## Pay Table B1

| Hand Type | \% Probability | Pays | \% Return |
| :--- | ---: | ---: | ---: | ---: |
| Straight Flush | 0.2172 | 40 | 8.6878 |
| 3 of a Kind | 0.2353 | 30 | 7.0588 |
| Straight | 3.2579 | 6 | 19.5475 |
| Flush | 4.9593 | 4 | 19.8371 |
| One Pair | 16.9412 | 1 | 16.9412 |
| Nothing | 74.3891 | -1 | -74.3891 |
| Total | 100.0000 |  | -2.3167 |

Pay Table B2

| Hand Type | \% Probability | Pays | \% Return |
| :--- | ---: | ---: | ---: |
| Straight Flush | 0.2172 | 40 | 8.6878 |
| 3 of a Kind | 0.2353 | 25 | 5.8824 |
| Straight | 3.2579 | 6 | 19.5475 |
| Flush | 4.9593 | 4 | 19.8371 |
| One Pair | 16.9412 | 1 | 16.9412 |
| Nothing | 74.3891 | -1 | -74.3891 |
| Total | 100.0000 |  | -3.4932 |

## Pay Table B3

| Hand Type | \% Probability | Pays | \% Return |
| :--- | ---: | ---: | ---: | ---: |
| Straight Flush | 0.2172 | 40 | 8.6878 |
| 3 of a Kind | 0.2353 | 30 | 7.0588 |
| Straight | 3.2579 | 6 | 19.5475 |
| Flush | 4.9593 | 3 | 14.8779 |
| One Pair | 16.9412 | 1 | 16.9412 |
| Nothing | 74.3891 | -1 | -74.3891 |
| Total | 100.0000 |  | -7.2760 |

The hit frequency for all three pay tables is $\mathbf{2 5 . 6 1 0 9 \%}$.

## Best 4 Out Of 5 Community Cards Side Bet

## Pay Table C1

| Hand Type | \% Probability | Pays | \% Return |
| :--- | ---: | ---: | ---: |
| Royal Flush | 0.007388 | 500 | 3.6938 |
| 4 of a Kind | 0.024010 | 100 | 2.4010 |
| Straight Flush | 0.072337 | 50 | 3.6168 |
| 3 of a kind | 2.256903 | 5 | 11.2845 |
| Flush | 4.410072 | 3 | 13.2302 |
| Straight | 3.917259 | 3 | 11.7518 |
| Two Pair | 4.753902 | 2 | 9.5078 |
| A Pair of Jacks O/B | 12.438360 | 1 | 12.4384 |
| Nothing | 72.119771 | -1 | -72.1198 |
| Total | 100.000000 |  | -4.1955 |

## Pay Table C2

| Hand Type | \% Probability | Pays | \% Return |
| :--- | ---: | ---: | ---: |
| Royal Flush | 0.007388 | 500 | 3.6938 |
| 4 of a Kind | 0.024010 | 100 | 2.4010 |
| Straight Flush | 0.072337 | 30 | 2.1701 |
| 3 of a kind | 2.256903 | 5 | 11.2845 |
| Flush | 4.410072 | 3 | 13.2302 |
| Straight | 3.917259 | 3 | 11.7518 |
| Two Pair | 4.753902 | 2 | 9.5078 |
| A Pair of Jacks O/B | 12.438360 | 1 | 12.4384 |
| Nothing | 72.119771 | -1 | -72.1198 |
| Total | 100.000000 | - | -5.6423 |

## Pay Table C3

| Hand Type | \% Probability | Pays | \% Return |
| :--- | ---: | ---: | ---: |
| Royal Flush | 0.007388 | 400 | 2.9550 |
| 4 of a Kind | 0.024010 | 80 | 1.9208 |
| Straight Flush | 0.072337 | 25 | 1.8084 |
| 3 of a kind | 2.256903 | 5 | 11.2845 |
| Flush | 4.410072 | 3 | 13.2302 |
| Straight | 3.917259 | 3 | 11.7518 |
| Two Pair | 4.753902 | 2 | 9.5078 |
| A Pair of Jacks O/B | 12.438360 | 1 | 12.4384 |
| Nothing | 72.119771 | -1 | -72.1198 |
| Total | 100.000000 |  | -7.2229 |

## Best 4 out of 5 Community Cards Side Bet Pay Table C4

| Hand Type | \% Probability | Pays | \% Return |
| :--- | ---: | ---: | ---: |
| Royal Flush | 0.007388 | 250 | 1.8469 |
| 4 of a Kind | 0.024010 | 100 | 2.4010 |
| Straight Flush | 0.072337 | 40 | 2.8935 |
| 3 of a kind | 2.256903 | 6 | 13.5414 |
| Flush | 4.410072 | 3 | 13.2302 |
| Straight | 3.917259 | 3 | 11.7518 |
| Two Pair | 4.753902 | 2 | 9.5078 |
| A Pair of Jacks O/B | 12.438360 | 1 | 12.4384 |
| Nothing | 72.119771 | -1 | -72.1198 |
| Total | 100.000000 |  | -4.5089 |


| Pay Table C5 |
| :--- |
| Hand Type \% Probability Pays \% Return <br> Royal Flush 0.007388 250 1.8469 <br> 4 of a Kind 0.024010 100 2.4010 <br> Straight Flush 0.072337 30 2.1701 <br> 3 of a kind 2.256903 6 13.5414 <br> Flush 4.410072 3 13.2302 <br> Straight 3.917259 3 11.7518 <br> Two Pair 4.753902 2 9.5078 <br> A Pair of Jacks O/B 12.438360 1 12.4384 <br> Nothing 72.119771 -1 -72.1198 <br> Total 100.000000  -5.2322 |

## Pay Table C6

| Hand Type | \% Probability | Pays | \% Return |
| :--- | ---: | ---: | ---: |
| Royal Flush | 0.007388 | 250 | 1.8469 |
| 4 of a Kind | 0.024010 | 100 | 2.4010 |
| Straight Flush | 0.072337 | 50 | 3.6168 |
| 3 of a kind | 2.256903 | 5 | 11.2845 |
| Flush | 4.410072 | 3 | 13.2302 |
| Straight | 3.917259 | 3 | 11.7518 |
| Two Pair | 4.753902 | 2 | 9.5078 |
| A Pair of Jacks O/B | 12.438360 | 1 | 12.4384 |
| Nothing | 72.119771 | -1 | -72.1198 |
| Total | 100.000000 |  | -6.0424 |

The hit frequency for the above six pay tables is $\mathbf{2 7 . 8 8 0 2 \%}$.


[^0]:    ${ }^{1}$ There are 25989605 -card hands but only 134459 of them are distinct. Also, it doesn't matter which two of the five cads are in the player's hand because there is no dealer hand.

